

COBE Experience With Filter QUEST*

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ABSTRACT

A gyro-based filter variation on the standard QUEST attitude determination algorithm is applied to the Cosmic Background Explorer (COBE). Filter QUEST is found to be three times as fast as the batch estimator and slightly more accurate than regular QUEST. Perhaps more important than its speed or accuracy is the fact that Filter QUEST can provide real-time attitude solutions when regular QUEST cannot due to lack of observability. Filter QUEST is also easy to use and adjust for the proper memory length. Suitable applications for Filter QUEST include coarse and real-time attitude determination.

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1. INTRODUCTION

Filter QUEST is a new version of the familiar QUEST attitude determination software. Because it makes use of gyro data, it promises to combine the speed of QUEST with the accuracy of gyro-based estimators. Just how fast and how accurate is seen from experience with Cosmic Background Explorer (COBE) data.

2. BACKGROUND ON QUEST

QUEST is Shuster's implementation of the Davenport q-algorithm for attitude determination (Reference 1). Since its introduction in 1978, it has been a standard part of the attitude ground support systems built for three-axis spacecraft at the NASA Goddard Space Flight Center Flight Dynamics Facility. Explanations of the algorithm and its implementation are available elsewhere (References 2, 3, and 4), but a few remarks about QUEST that are relevant to the following discussion are included for convenience.

First, QUEST uses only unit vector observations expressed in body coordinates. These are denoted here by \hat{w}_i , where i indicates the particular observation. The corresponding reference vectors in inertial coordinates are \hat{v}_i . Each observation is also given a scalar weight a_i , which is normalized so that the sum over all observations equals one.

$$\sum_{i=1}^N a_i = 1 \quad (1)$$

Second, an intermediate quantity called the attitude profile, or "B-matrix," is computed as follows:

$$B = \sum_{i=1}^N a_i \hat{v}_i \hat{w}_i^T \quad (2)$$

Several secondary quantities are also defined:

$$z \equiv \begin{bmatrix} B_{23} - B_{32} \\ B_{31} - B_{13} \\ B_{12} - B_{21} \end{bmatrix} \quad (3)$$

$$S \equiv B + B^T \quad (4)$$

$$\sigma \equiv \text{trace}[B] \quad (5)$$

Third, the attitude quaternion q is then the solution to the eigenvalue equation

$$Kq = \lambda q \quad (6)$$

where the matrix is constructed from those secondary quantities:

$$K = \begin{bmatrix} (S - \sigma I) & z \\ z^T & \sigma \end{bmatrix} \quad (7)$$

3. FILTER QUEST

Filter QUEST is a natural generalization of the QUEST attitude determination algorithm (References 5 and 6). QUEST is limited to using "single-frame" observations from the same time or a constant attitude. Filter QUEST, however, can accept observations from different times or attitudes so long as it has the incremental rotation angle between those times. This information typically comes from gyros. In the past, systems have been built that propagate the unit vector observations to a common time before providing them to QUEST for "batch" processing. Filter QUEST differs in that it handles that preprocessing internally. Filter QUEST also differs in that it is a sequential estimator with a simple "fading" memory rather than a batch estimator. In Filter QUEST, the B-matrix is corrected for spacecraft rotation between observation times t_i and t_{i-1} . Here $B_{i/j}$ denotes the value of the B-matrix at time t_i based on observations up to and including those at time t_j :

$$B_{i/i-1} = \alpha \Phi(t_i, t_{i-1}) B_{i-1/i-1} \quad (8)$$

The memory length parameter α is a number between zero and one. α equal to one implies infinite memory, and α equal to zero produces the standard QUEST algorithm. Φ is the attitude propagation matrix computed from the angular increment vector $\vec{\theta}$ for the time interval (t_i, t_{i-1}) . It is given by the expression

$$\Phi(t_i, t_{i-1}) = \cos \theta I + (1 - \cos \theta) \hat{\theta} \hat{\theta}^T - \sin \theta [\hat{\theta}_\times] \quad (9)$$

where θ is the magnitude and $\hat{\theta}$ is the direction of $\vec{\theta}$

$$\vec{\theta} = \theta \hat{\theta}$$

and

$$[\hat{\theta}_x] = \begin{bmatrix} 0 & -\hat{\theta}_3 & \hat{\theta}_2 \\ \hat{\theta}_3 & 0 & -\hat{\theta}_1 \\ -\hat{\theta}_2 & \hat{\theta}_1 & 0 \end{bmatrix}$$

At each observation time, the B-matrix is updated using the new pair of vectors \hat{v}_i and \hat{w}_i

$$B_{i/i} = B_{i/i-1} + a_i \hat{v}_i \hat{w}_i^T \quad (10)$$

Attitude is computed at selected times using the standard QUEST algorithm.

There are three reasons for interest in Filter QUEST. First, because it uses gyro data, it can provide more accurate real-time attitude solutions than regular QUEST. It can also extend observability into periods such as orbit night, when single-frame methods may be useless. Second, because QUEST is fast, it is hoped that Filter QUEST will provide accuracies approaching those of the batch estimator with less CPU time. This is useful for producing 24-hour attitude histories, as in definitive processing. Third, the simplicity and familiarity of Filter QUEST make it a suitable "training" filter. Tuning is accomplished by adjusting the single parameter α , which can always be set equal to zero to return to the original QUEST. Because QUEST is so well known, any improvement is also likely to be accepted.

To evaluate its performance with real data, Filter QUEST was built into the Coarse Attitude Determination Subsystem (CADS) of the Cosmic Background Explorer (COBE) Flight Dynamics Support System (FDSS), where it replaced regular QUEST. Tests were then made to see how much attitude error could be eliminated by increasing α and to determine an optimal value for α that would minimize error while avoiding long-term divergence. For these tests, the Fine Attitude Determination Subsystem (FADS) solution computed using Diffuse Infrared Background Experiment (DIRBE) star observations was used as a reference. FADS is a batch estimator that uses gyro data to provide the most accurate COBE attitude solutions. The Filter QUEST and these DIRBE batch solutions were then compared in the Quality Assurance (QA) subsystem, where the plots and statistics on their differences were computed. Finally, timing comparisons were made for the batch, regular QUEST, and Filter QUEST estimators.

4. THE EFFECT OF MEMORY LENGTH

The length of the fading memory is controlled by the parameter α , which determines the effect of the current observation on the current attitude estimate. Although α is constant, the

“effective” weight, which is the ratio of the current observation weight to the total weight, is not. If all observation types have the same weight a , the sum of these weights at time t_i is

$$W_i = a \sum_{j=0}^{i-1} a^j = \frac{a(i - a^i)}{1 - a} \quad (11)$$

The ratio of the current weight to the total weight is

$$1 \geq \frac{a}{W_i} = \frac{1 - a}{(1 - a^i)} \rightarrow 1 - a \quad (12)$$

This number decreases from one and converges to $(1 - a)$. Thus, the fading memory filter is more responsive to early observations than later ones. This is particularly so a is set high and there is no a priori solution. In this case, the solution can lock up quickly based on a few inaccurate measurements and then take a very long time to converge to the correct attitude.

The error in a Filter QUEST solution is taken to be the difference between that solution and a DIRBE batch solution. For the short timespan shown in Figure 1, the error is random with amplitude consistent with the 0.5 degree Sun sensor resolution. The size of the error corresponds to the spread of the difference curve or the root of the mean squared (rms) difference.

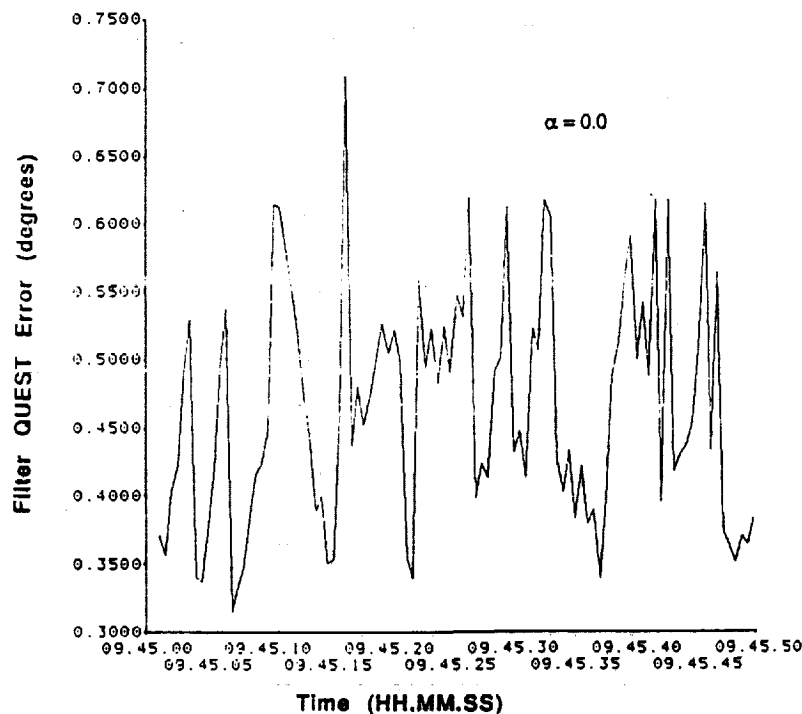


Figure 1. Filter QUEST Solution Error

As the memory length parameter α increases from zero to one, the rms difference decreases, as shown in Figure 2. The rms difference does not go to zero because the observation errors are not completely random.

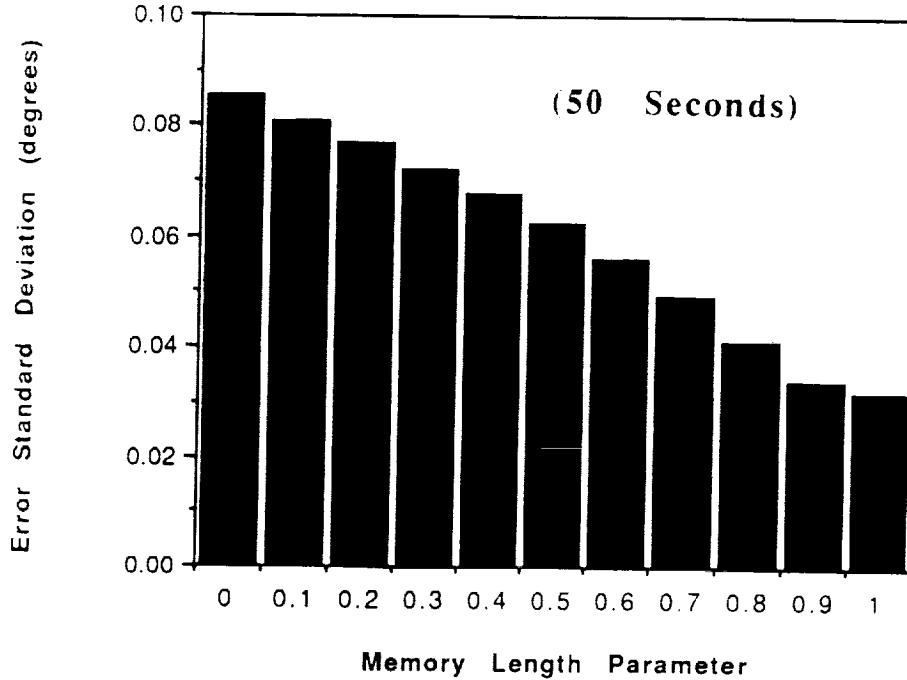


Figure 2. Error Standard Deviation as a Function of Memory Length

It would seem then that the memory length should be made as large as possible. The adverse effect of doing so becomes apparent over longer timespans. As shown in Figure 3, the difference can increase with time and exceed the one degree (3σ) COBE accuracy goal. Figure 4 gives a plot of the rms differences for full-orbit (100-minute) solutions as a function of α . There is no sharply defined value of α that minimizes the error. Any value between 0.90 and 0.99 is suitable. The determination of this value is sometimes called "tuning." In practice, the true attitude is not apt to be known, and tuning must be done by examining how well the observations are fit by the attitude solution. QUEST and Filter QUEST provide such a figure of merit in the output "RESIDU" ϱ , which equals one minus the "overlap eigenvalue" λ (Reference 7). When all the observations fit the attitude solution, RESIDU equals zero.

$$\varrho = 1 - \lambda \quad (13)$$

RESIDU is plotted in Figure 5 for the solution of Figure 3. Just as the growing difference curve indicates that α is set too high, the growth of RESIDU says the same thing without the need for an absolute attitude reference. Figure 6 gives a plot of the maximum RESIDU values for full-orbit solutions as a function of α corresponding to the rms differences provided by Figure 4. Again, there is no clearly optimal value for α . Any value between 0.90 and 0.99 will do. Alternatively, α can be chosen such that RESIDU is as large as possible but does not grow over time.

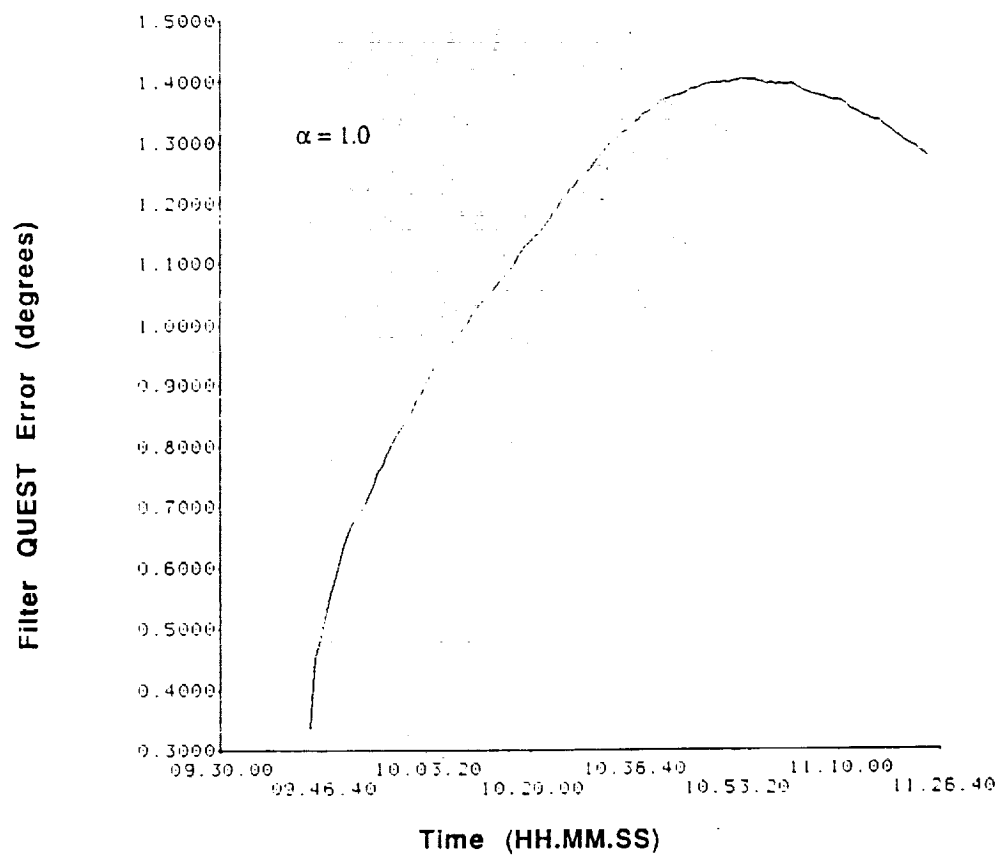


Figure 3. Divergence of Filter QUEST Solution Over a Full Orbit

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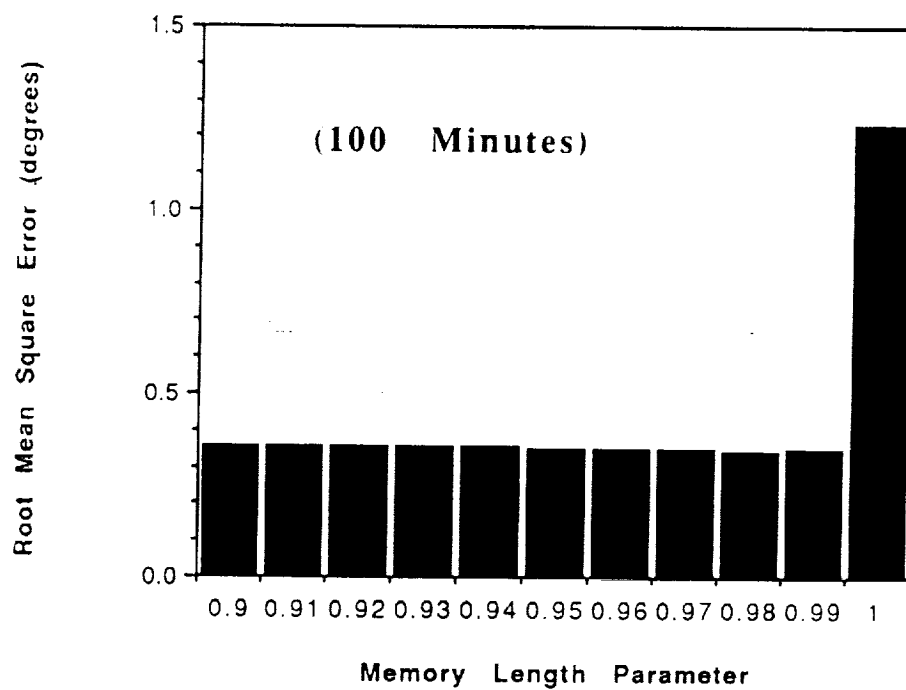


Figure 4. Root Mean Square Error as a Function of Memory Length

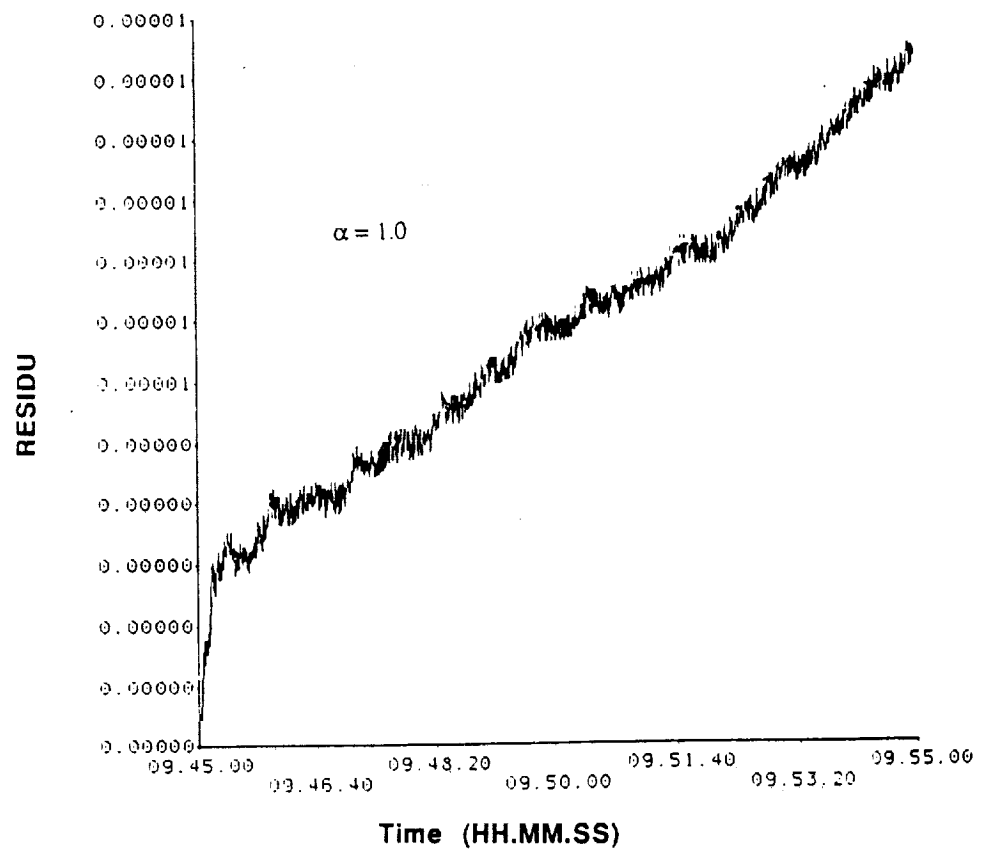


Figure 5. RESIDU as an Indicator of Divergence

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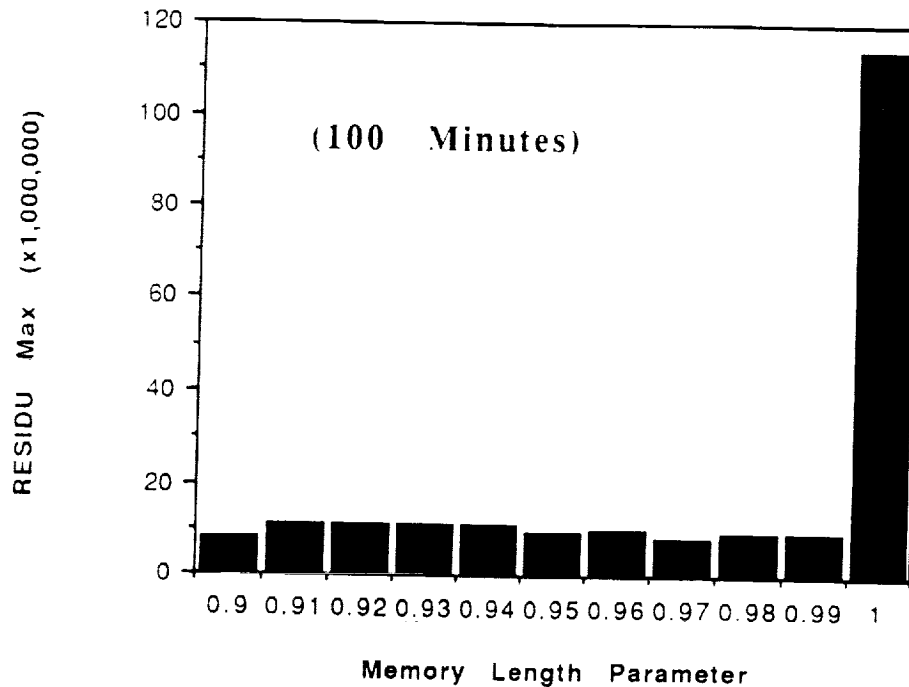


Figure 6. RESIDU as a Function of Memory Length

5. SPEED AND ACCURACY COMPARISONS

Since the batch estimator and QUEST represent the current COBE capabilities, they are used as standards against which Filter QUEST is measured. For comparison, all three estimators are applied to a full orbit (100 minutes) of data. The memory length parameter α is set to 0.98 to keep the attitude from diverging over this timespan. As before, the reference is a batch solution computed using DIRBE star data. The batch error plotted here is the difference between this reference and a batch solution obtained from Sun and Earth sensor data alone.

The rms errors of the three solutions are shown in Figure 7. For this comparison, gyro biases were first solved for in the batch estimator, and the corrections were made in the Data Adjuster (DA) subsystem. This improved the Filter QUEST solution accuracy as well as that of the batch estimator. In spite of having gyro data, Filter QUEST had an rms error only 14 percent smaller than that of regular QUEST, and still far greater than that of the batch solution. The reason for this small improvement is that like any sequential estimator, Filter QUEST cannot look ahead of the current solution time. While the batch estimator effectively has infinite memory and thus can average out orbital and higher frequency errors, Filter QUEST cannot set α to one without diverging. Unless observation errors are really random, or are much smaller than the propagation error, no filter compares favorably with a batch estimator in accuracy. Unfortunately, all nonrandom errors seem not to have been eliminated from the COBE observations.

The speed of each estimator in producing these solutions is plotted in Figure 8. These numbers are for the Flight Dynamics Facility Hitachi NAS 8063 computers and come from

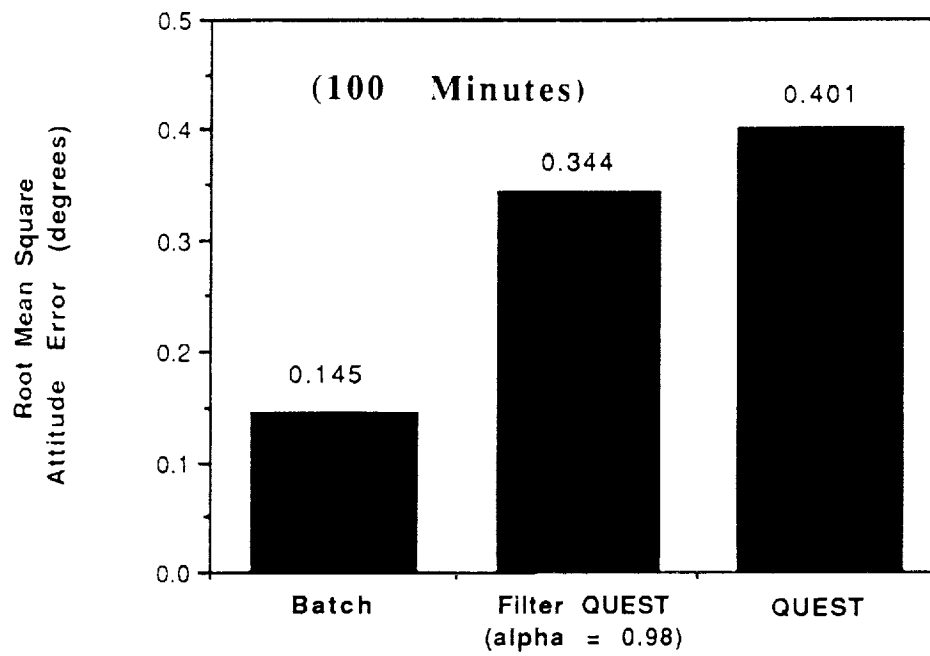


Figure 7. Estimator Error

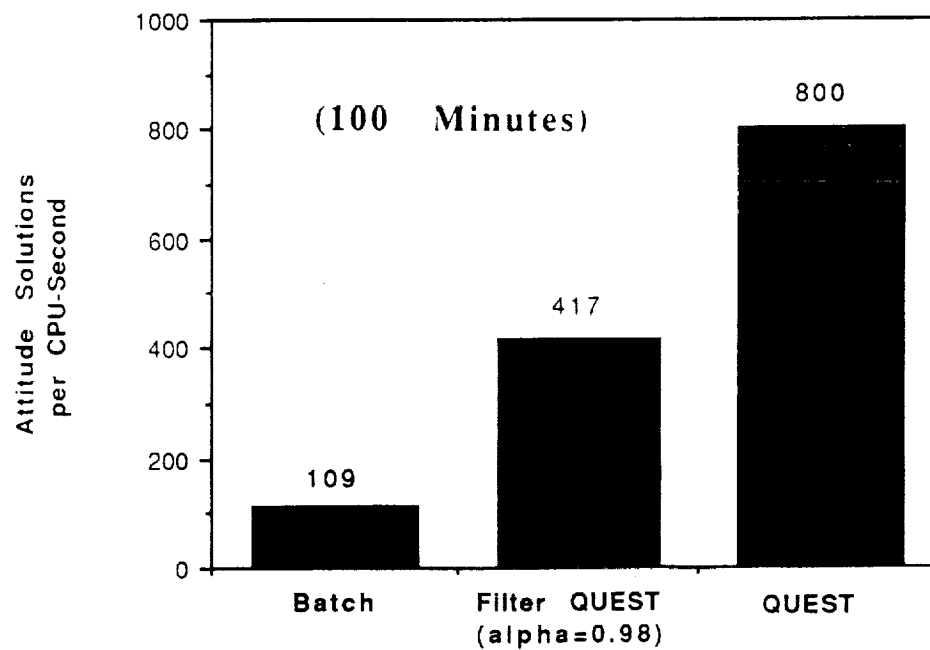


Figure 8. Estimator Speed

running only the estimators. The conversion of telemetry, correction for known calibration errors, and generation reference data are not included. These speeds are for running interactively rather than in background mode. Background execution can be much faster, but the relative speeds should be similar. As expected, regular QUEST is the fastest. Addition of propagation to Filter QUEST cuts its speed in half. It is still, however, more than three times as fast as the batch estimator, the speed of which has been greatly increased since it was originally coded.

6. CONCLUSIONS

In conclusion, this study suggests that Filter QUEST pays in speed for what can be a small increase in accuracy. For this reason, it seems more suited for use as an upgrade from regular QUEST than as a replacement for the batch estimator. For spacecraft with only limited periods of complete attitude observability, however, this can mean the difference between knowing and not knowing the attitude in real time. Because the Filter QUEST algorithm reduces to regular QUEST when the memory length parameter α is zero, there is no risk in substituting it in all current and future applications of QUEST.

7. ACKNOWLEDGMENTS

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